

# Cognitive Capacity Constraint and Attention Allocation in Human Decision Making

Qi Wu<sup>1,\*</sup>, Tetsuya Shimokawa<sup>2</sup>

<sup>1</sup>Graduate School of Management, Tokyo University of Science, Tokyo, Japan

<sup>2</sup>Department of Business Economics, Tokyo University of Science, Tokyo, Japan

## Email address:

shkidwq@hotmail.com (Qi Wu), simokawa@rs.tus.ac.jp (Tetsuya Shimokawa)

\*Corresponding author

## To cite this article:

Qi Wu, Tetsuya Shimokawa. Cognitive Capacity Constraint and Attention Allocation in Human Decision Making. *Journal of Finance and Accounting*. Vol. 10, No. 2, 2022, pp. 141-150. doi: 10.11648/j.jfa.20221002.17

**Received:** March 16, 2022; **Accepted:** April 11, 2022; **Published:** April 20, 2022

---

**Abstract:** The Rational Inattention (RI) model has attracted attention in recent years as a promising candidate for modeling bounded rationality in the fields of decision making and game theory research. The model assumes that there is a cognitive cost (cost of information processing) that is proportional to the amount of mutual information obtained from signals, thereby making it possible to explain various phenomena observed in the market at a certain level. However, the RI model still lacks a sufficient cognitive foundation. In this study, we conducted an experiment to examine whether the cognitive costs and constraints on information processing, which are the assumptions of the Rational Inattention Model, are reasonable from the perspective of neuroeconomics using biometric data such as gaze information and brain responses. We adopted the sequential investment task with a view to applying it to finance. Our results showed that the stochastic choice rational inattention model fit the behavioral data of the present experiment, the larger the cognitive cost the more activated the brain regions involved in costly cognition, And the consistency between gaze information and the capacity constraint of the Kalman filter type model, as expected, when there is a lot of information, not all information can be processed, so more accurate decisions cannot be made.

**Keywords:** Rational Inattention, Mutual Information, Experiment

---

## 1. Introduction

In economic and financial theory models have been constructed under the assumption of rational agents. Recently, the Rational Inattention (RI) model has been attracting attention as a model of bounded rationality that is closer to reality. This model limits human rationality by measuring the information obtained from signals in terms of the amount of mutual information, setting a limit on the cognitive ability to process the information, and assuming a cognitive cost (the cost of information processing) that is proportional to the amount of mutual information. This model has been widely applied and has been reported to explain various phenomena observed in real markets, such as finance, auctions, market prices, policy analysis, and labor employment.

The history of the RI models begins with Sims, and there have been two major streams of research. The first is the Kalman filter type model initiated by Sims [1, 3]. This model

is used in dynamic environments, where the accuracy of the Kalman filter depends on the subject's ability to process information. The other stream is the stochastic choice type model [2]. In this model, information processing costs are proportional to the amount of mutual information, and the subject decides which information and actions to use to maximize the expected utility. This model is also known to be closely related to the logit-type stochastic choice model and recently been applied to various fields, including game theory [4, 12, 13], dynamic modeling [14, 16, 18, 19] and information acquisition processing costs [11, 15, 17, 20].

However, contrary to the spread of the theory, there has been no progress in testing the validity of the cognitive assumptions underlying the RI model. In this study, the validity of this method was verified using brain and gaze information.

Specifically, we will examine whether the cognitive costs and constraints on information processing, which are the assumptions of the Rational Inattention Model, are reasonable from the perspective of neuroeconomics using biometric data.

We adopted the sequential investment task to apply it to finance.

- (1) Examining the consistency between behavior, changes in cerebral blood oxidized hemoglobin concentration in the dorsolateral prefrontal cortex, and Stochastic choice type model information cost parameter.
- (2) Examining the consistency of gaze information and capacity constraints of the KF model.

## 2. Methods

### 2.1. Sequential Investment Task

In this task, participants make predictions about price sequence returns each period and decide whether to invest in a single stock or a safe asset. The participant's goal is to maximize the expected return. As information, the participants are presented with up to eight stocks whose returns are

correlated with those of the target stock. The stocks used are all from the Shanghai Stock Exchange and the target stocks used are Henan Taloph Pharmaceutical Stock Co., Ltd. (600222), Air China (601111) and China Construction Bank (601939) for the period December 26, 2020 to October 15, 2021. To intentionally create a difference in the amount of mutual information obtained, the three treatments are divided, and one participant does three experiments, e.g., the first experiment has only one signal of target stock, the second has four signals of target stock, and the third has Eight. If the estimated return is higher than the safe asset interest rate (0 in this case), they invest fully in the stock, otherwise, they do not. However, calculating each period's return from the signals requires a certain amount of computation, so given the cognitive constraints, participants may not reasonably estimate. The price paths used in this experiment are shown in Figure (Price sequence).

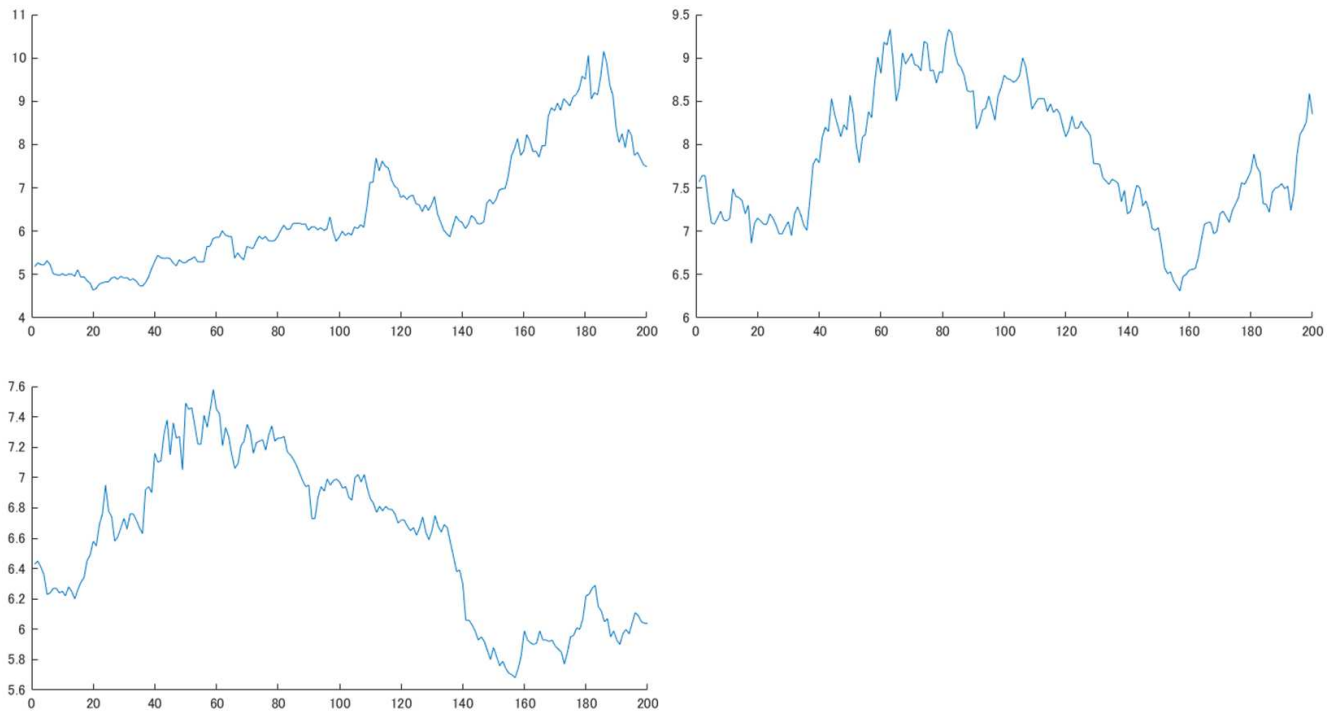


Figure 1. Price sequence.

This time we use capital asset pricing model (CAPM) to define the relationship between the target stock and the signal stocks. Let  $r_M$  be the return of the market portfolio as target stock and be constant over time. The relationship between the signal stock and the target stock is  $r_{si} = \beta_i r_M + e_i$ , where  $e_i \sim N(0, \sigma_N^2)$  i.i.d. .  $\beta$  is the sensitivity of the expected excess asset returns to the expected excess market returns. The return on the safe asset is always assumed to be  $r_f = 0$ . The target stock price sequence presented to the participant's GUI is calculated as  $r_M = \frac{r_{si} - e_i}{\beta_i}$ .

Figure 2 shows the experimental graphical user interface

(GUI) that participants faced during the decision-making process. To avoid biometric artifacts caused by the button selection behavior, the experiment used a cylinder-type input to allow the investment rate to be varied continuously. However, following the model setup, the investment rate of each participant was converted in the analysis to binary based on the time-averaged investment rate.

### 2.2. Experimental Setup

The biometric information used in this analysis was the change in blood hemoglobin concentration in the prefrontal area. Functional NIRS (BriteMKII supplied by Artinis Medical Systems) was used to measure the blood hemoglobin concentration in the prefrontal area. Figure 3 shows the brain

regions that we focused on in this study. We will focus on the dorsolateral, ventral, and rostral regions, which are considered to be closely related to costly cognition, working memory, and reasoning [4-7]. The prefrontal cortex may be roughly divided into the orbitofrontal cortex ((Brodmann Areas [BA]) 11, 12, and 13), medial prefrontal cortex ((Brodmann Areas [BA]) 24, 25, 32, and mesial portions of 10), and dorsolateral cortex ((Brodmann Areas [BA]) 8, 9, and 46). Each region has a distinct cytoarchitecture and function as well as distinct connections. Briefly, the orbitofrontal cortex is involved in decision making, processing award and punishment; and the medial prefrontal cortex, particularly the anterior cingulate cortex, mediates emotional monitoring and self-regulation [10]. The dorsolateral prefrontal cortex (including Brodmann Areas 46, 9) is involved in working memory. Working

memory is the ability to hold a limited amount of information in mind for a short period. For example, working memory is necessary for holding a phone number 'in mind', or keeping track of geographical locations as someone gives you multistep directions to a location across town. This type of memory is critical to bridging temporal gaps so that the information can be 'worked' with or mentally manipulated for a short period of time. This ability to hold representations in mind is critical to other complex cognitive functions, such as decision-making, planning, and problem-solving [8]. Area 8A can be considered as a key area for the top-down control of attentional selection [9], it is also a very important region for this experiment, but this time we used NIRS as experimental equipment, it was difficult to measure the Area 8A.

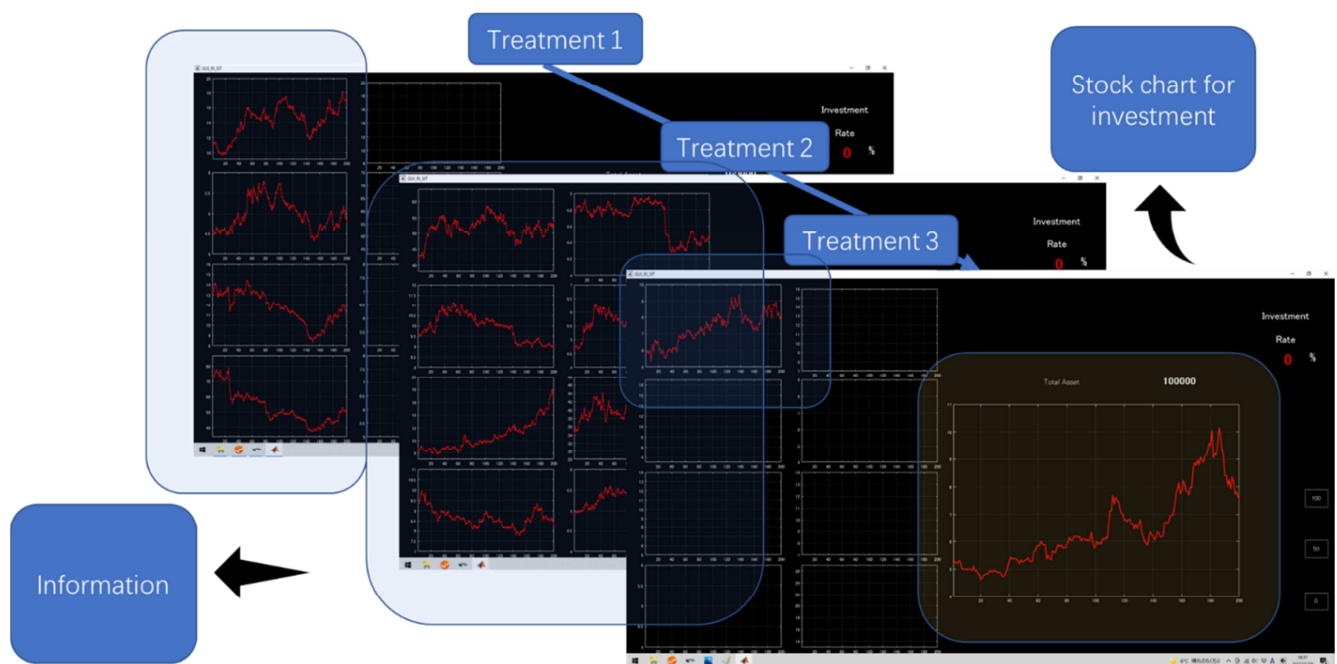


Figure 2. GUI image.



Figure 3. Brain regions of interest.

The positions of 19, 21 and 22 in Figure 3 represent Dorsolateral area, Ventral-extralateral area, and Rostral area.

At the same time, we used gaze information to clarify what kind of signals the subject uses.

We used Tobii Pro X3-120 of Tobii Technology Co. If the subject were rational, it would have observed all the information. However, when there are many signals available, the subject is unable to take advantage of them, and the results show inattention to information. If we interpret the results using a Kalman filter type model, we observe that there is a capacity constraint in the gaze information.

### 2.3. Participants

The participants of this experiment were 14 undergraduate and graduate students of the School of Business, Tokyo University of Science. None of them had any investment experience. Subjects made 170 investment decisions for one price sequence and were rewarded up to 1000 yen (including 500 yen for participation) according to their points. Subjects were randomly assigned to a task

for each treatment. Subjects were briefed on the GUI operation and the investment task setting and then engaged in the task. Because of the epidemic, not enough samples were obtained for this experiment, and we would like to collect more samples in the future to improve the statistical study.

#### 2.4. The Kalman Filter Type RI Model

In the Kalman filter type RI model, the subject uses the Kalman filter to remove the observation noise from the observed signal (in this case, the actual observed stock return) and predicts the fundamental return (state) of the stock to be invested. In this case, the accuracy of the prediction is limited by the information processing capacity of the subject, which is called capacity in analogy to Shannon's term.

When the signal is one-dimensional, it can be modeled as follows. If the variance of the normal noise appearing in the observation equation is  $\sigma_N^2$  and the variance of the fundamental return appearing in the state update equation in period  $t$  is  $\sigma_t^2$ , the conditional mutual information quantity of the fundamental return  $f$  observed in each period  $t$  can be calculated as  $I_t(f) = \frac{1}{2} \log(1 + \sigma_N^{-2} \sigma_t^2)$ . If the limit of the amount of mutual information that can be used by the subject is  $\kappa$  due to the constraint of information processing capacity,  $\kappa = \frac{1}{2} \log(1 + \sigma_N^{-2} \sigma_{t-1}^2)$  can be established, and if this equation is solved for the variance of observation noise, it becomes  $\sigma_N^2 = \sigma_{t-1}^2 / (\exp(2\kappa) - 1)$ . In this model, unlike the usual Kalman filter discussion, the noise appearing in the observation equation is subjective; if the capacity  $\kappa$  is  $\infty$ , then the variance of the observation noise is 0, and if  $\kappa$  is 0, then the variance of the observation noise is  $\infty$ . In other words, the accuracy of the observed signal depends on the information processing capacity of the subject. Under this subjective observation noise, people invest according to

their earnings forecasts. Given the subjective observation noise, the update of people's expectations follows;

$$\mu_{t+1} = \mu_t + KG(\kappa)(r_s - \mu_{rs})$$

KG is the Kalman gain, a function of  $\kappa$ ,  $r_s$  is given as a signal of return,  $\mu_{rs}$  is the expected value of the signal return.

#### 2.5. Stochastic Choice Type RI Model

In the stochastic choice RI model, the subject is assumed to make decisions in two steps. In the first step, the subject selects an information strategy on which signals to use, and in the second step, the subject selects an action that optimizes the expected profit according to the prediction of the fundamental return from the information. To identify the fundamental return, it is optimal to use as many and as informative signals as possible, but a certain percentage of the cognitive cost  $\lambda$  is required in proportion to the amount of mutual information obtained from the signals.

Specifically, in the second stage, the subject decides whether to invest in stock or safe assets according to the conditional distribution  $p(r_M | S)$  of the Market portfolio return  $r_M$  under the assumption that the signal  $S$  is used. Since there are eight signals at this time, so  $S = s_1, s_2, \dots, s_8$ . In the first stage of information strategy selection, given this expected utility, the subject decides which signal structure  $p(r_M | S)$  is preferable. However, to obtain more detailed information, the cost  $\lambda \{H(r_M) - H(r_M | S)\}$  is incurred according to the amount of mutual information. Here,  $H$  is the Shannon entropy, and the mutual information is defined as the decrease in entropy due to the acquisition of the signal.  $\lambda$  is a parameter that represents the cognitive cost per unit of mutual information, which is zero for rational subjects. Under such a setup, the first stage problem can be written as follows:

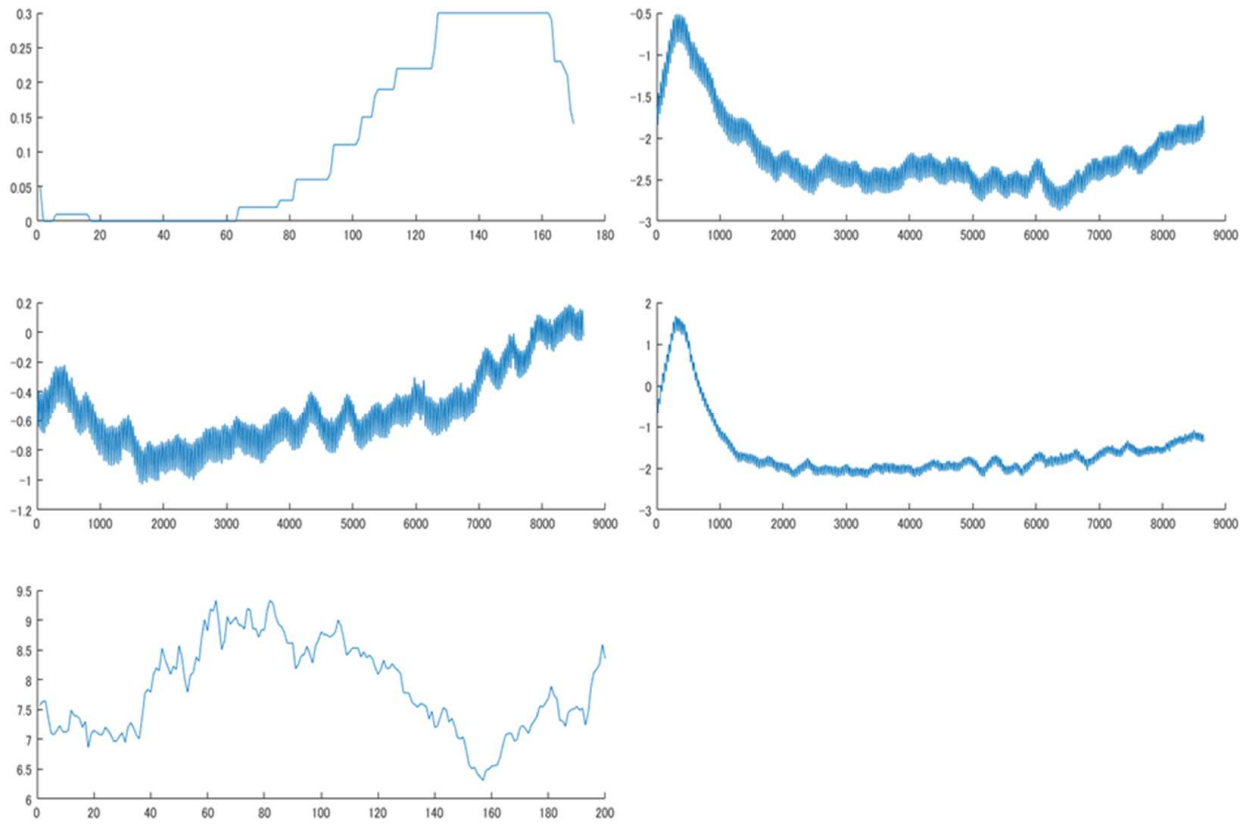
$$\max_{p(S|r_M)} \sum_{r_M} \sum_S V(p(r_M | S)) p(S | r_M) p(r_M) - \lambda \{H(r_M) - H(r_M | S)\}$$

The solution to this problem can be derived by the following softmax-type choice rule

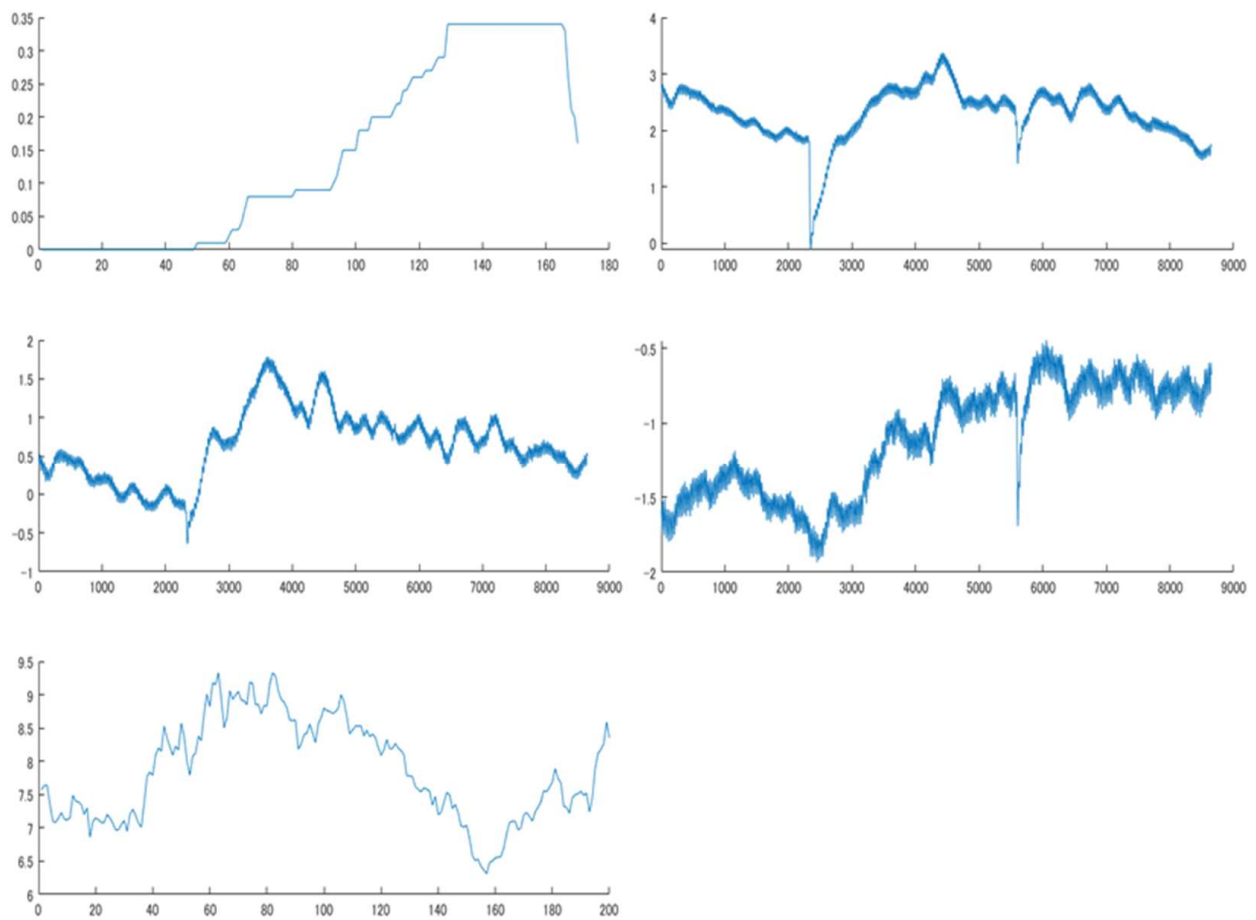
$$\pi_t = \frac{\exp(\frac{r_{Mt}}{\lambda})}{\exp(\frac{r_{ft}}{\lambda}) + \exp(\frac{r_{Mt}}{\lambda})} = \frac{\exp(\frac{r_{Mt}}{\lambda})}{1 + \exp(\frac{r_{Mt}}{\lambda})}, \forall t$$

The return of the safe asset  $r_f$  is set to zero in this experiment so  $\exp(\frac{r_{ft}}{\lambda}) = 1$ . If the cost is infinite,  $\pi_t = \frac{1}{2}$  the subject will be a person who chooses completely randomly.

Assuming a hyper-rational person with no information cost, he will gather information and choose the option with the lower payoff with probability 0, or the option with the higher payoff with probability 1.



**Figure 4.** Price series, investment rate, NIRS (Participant A).



**Figure 5.** Price series, investment rate, NIRS (Participant B).

The difference between the Rational case and the RI case, in the rational case, the stochastic choice type model is not related to the cost. Kalman filter type model has all the information available. In the RI case, the stochastic choice type model has the information cost. and the Kalman filter type model has a capacity constraint.

### 3. Empirical Results

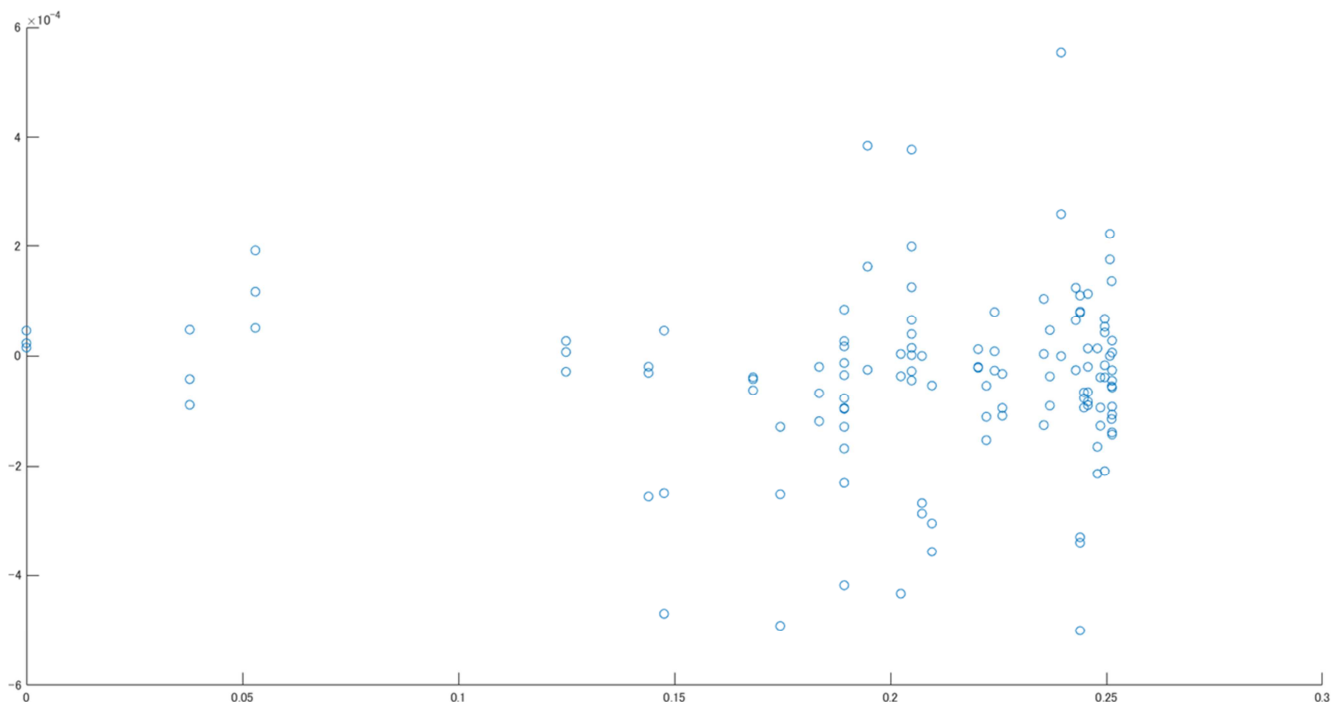
#### 3.1. Validation of the Stochastic Choice Model Using Cerebral Blood Hemoglobin Concentration

First, let's take a look at the time series of observed data. Figure 4 and Figure 5 shows a representative sequence of the price sequence (lower left), investment rate (upper left), and changes in blood hemoglobin concentration (oxidized blood hemoglobin concentration - deoxidized blood hemoglobin concentration) in the rostral (middle right), Dorsolateral (middle left), and Ventral-extra lateral (upper

right) regions.

In the analysis, we first examine the relationship between the variance of the investment rate and the hemoglobin concentration in blood, both when the cognitive cost  $\lambda$  are larger in the Stochastic choice type RI model and when the capacity  $\kappa$  is smaller in the Kalman filter type RI model the variance of the investment rate is larger.

The horizontal axis of this figure represents the variance of the investment rate, and the vertical axis is represented by the hemoglobin concentration in blood. Each circle is one path of one subject which you will observe a rightward trend. In other words, it is conceivable that  $\lambda$  hemoglobin concentration may be positively correlated. When the variance of the investment rate increases, the  $\lambda$  increases, and when the  $\lambda$  is large, the hemoglobin concentration in the blood is large, which means that the hemoglobin concentration is activated and the information processing is going on.



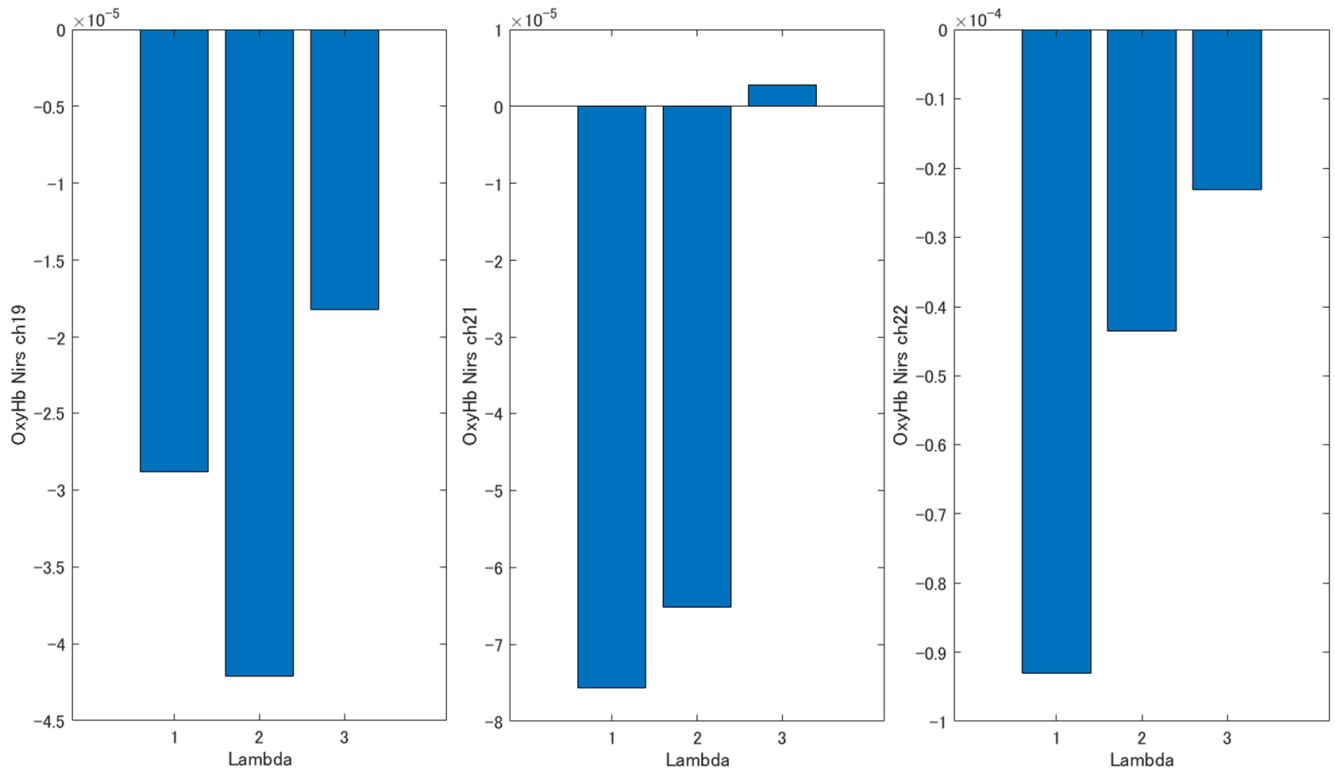
**Figure 6.** Relationship between the variance of investment rate and blood hemoglobin concentration.

We examined the relationship between the parameter values of each RI model and the hemoglobin concentration in the blood. As mentioned earlier, the amount of mutual information obtained from signals is determined subjectively, and this cannot be observed directly. The parameters of each model can be considered as its alternative indicators. Figure 7 shows the average hemoglobin concentration in blood of each group in the trichotomous analysis. The figure shows the hemoglobin concentration in the blood of each sample, divided into three groups according to the size of the estimated model

parameters (the left bar shows a small value group and the right bar is a large value group). From left to right, they correspond to rostral, dorsal-extra lateral, and ventral-extra lateral regions.

From this figure, we can see that brain activity is consistent with the assumptions of the RI models except rostral area. In other words, the larger the cognitive cost  $\lambda$  the more activated the brain regions involved in costly cognition. However, not strongly statistically significant was found at this time, we will collect more samples sufficient for the statistical study.



Figure 7. Trichotomous analysis of  $\lambda$ .

### 3.2. Validation of a Kalman Filter-Type Model Using Gaze Data

Next, we examine the Kalman filter type RI model by Gaze information. Tables 1, 2, and 3 show the percentage of eyes staying on each signal at eight, four, and one signals, respectively, and Figure 8 shows the signal positions. Table 4 is the correlation coefficient and Kalman Gain without capacity constraint. From these tables, we can see in the case of one signal, the share of the total time is 91.88%, definitely

looking at the signal. In the case of four signals, Signal 2 is often seen, and Signal 4 is not seen very often. The correlation coefficient below shows that Signal 2 is the largest and Signal 4 is the smallest, which means that those with no correlation are not watched by humans in decision-making. Next, in the case of Signal 8, unlike the previous case, Signal 1, Signal 4, and Signal 8 are rarely looked at, which means that information is selected based on capacity constraints. In the case of signal 8, we can see that there is a correlation, but this correlation can hardly be processed by humans.

Table 1. Gaze Information (8 Signals case).

	Signal1	Signal2	Signal3	Signal4	Signal5	Signal6	Signal7	Signal8
Share of Total Time (%)	6.32	9.91	13.56	3.48	13.85	28.66	17.26	6.95

Table 2. Gaze Information (4 Signals case).

	Signal1	Signal2	Signal3	Signal4
Share of Total Time (%)	14.33	39.28	15.09	10.18

Table 3. Gaze Information (1 Signal case).

	Signal1
Share of Total Time (%)	91.88

Table 4. Correlation coefficient and Kalman Gain.

Signals	Signal1	Signal4	Signal8										
Correlation Coefficient	0.3685	0.5085	0.8537	0.4422	0.0475	0.6963	0.8861	0.5642	0.7147	0.6405	0.715	0.7232	-0.8708
KG	0.0247	0.0118	1.3208	0.2287	-0.02	-0.2934	0.2412	0.2644	-0.2685	-0.1427	0.13653	0.14443	-0.1702

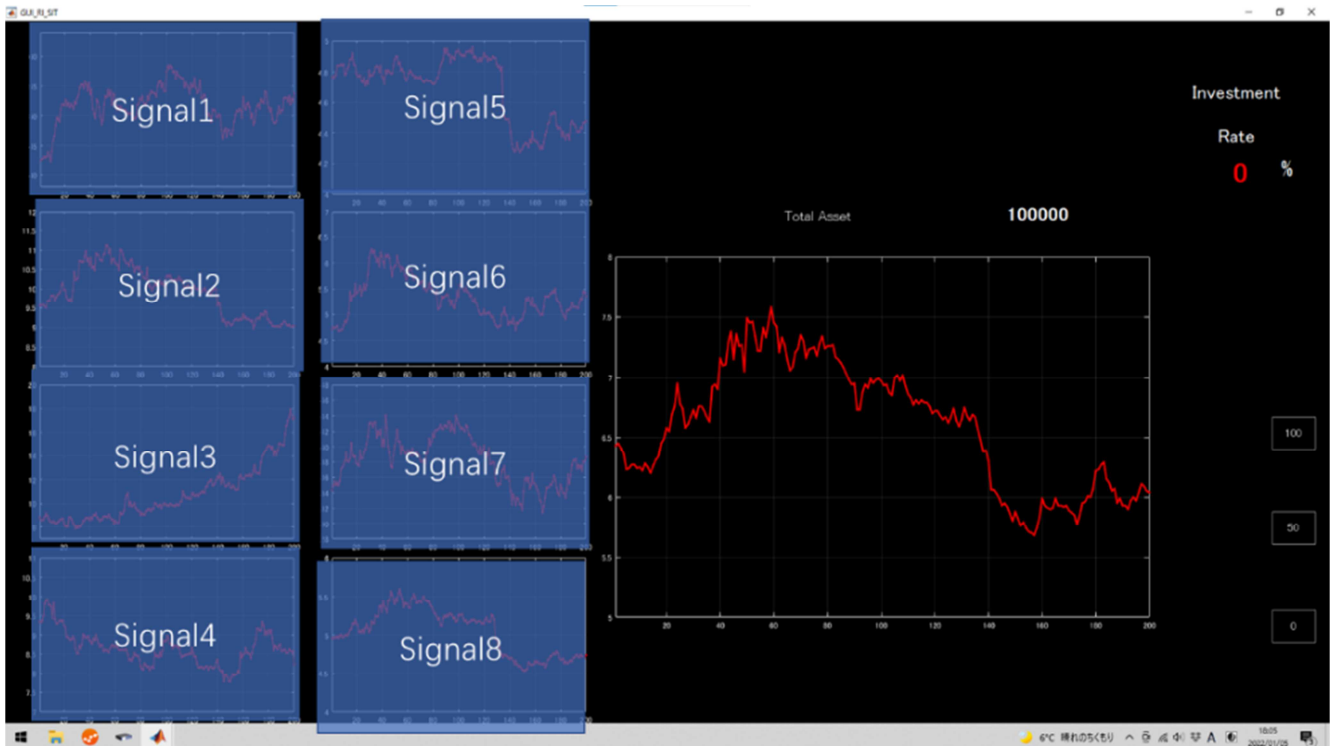


Figure 8. Signal positions.

Next, from the Heat map, Figures 9, 10, 11, we can see in the case of one signal, definitely looking at the signal. When there are eight signals, there will be signals that are rarely seen because of capacity constraints. In the case of Rational,

all signals should be seen, while in the case of Rational Inattention, cognitive constraints are in place, so the choice of information is happening.

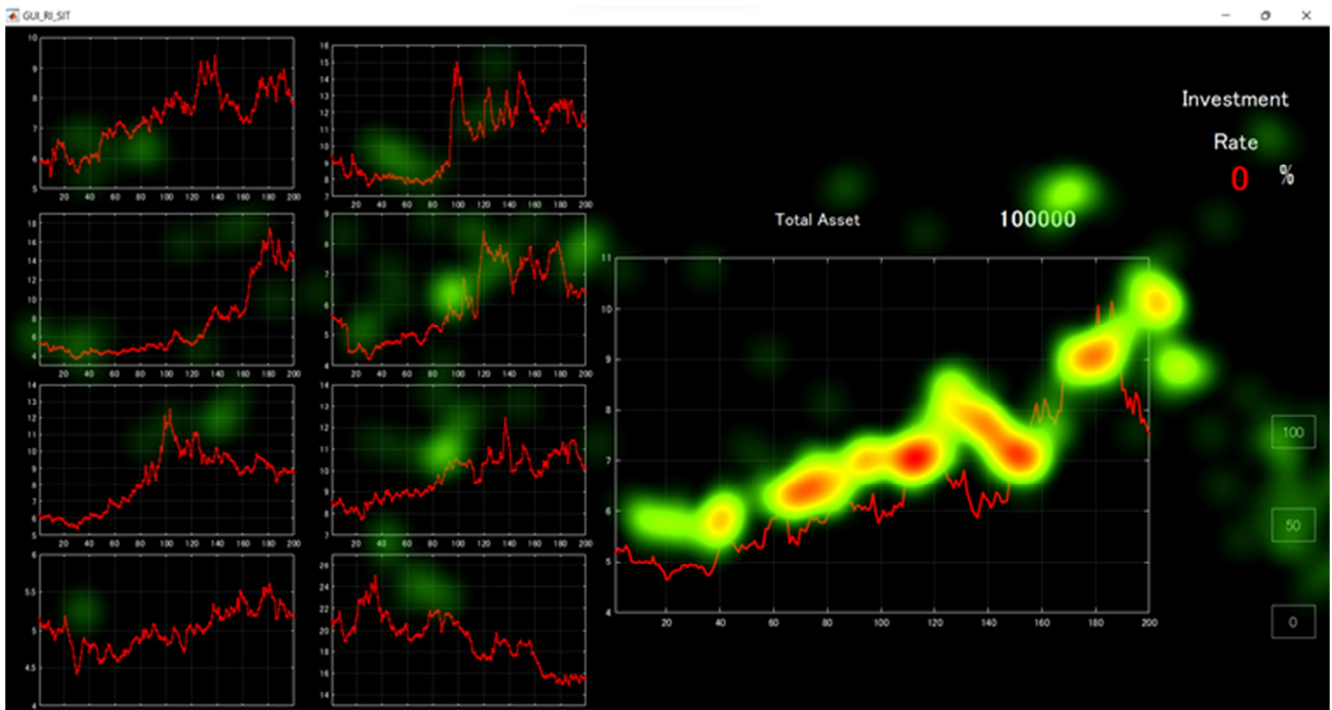


Figure 9. Heat map signal 8 case.



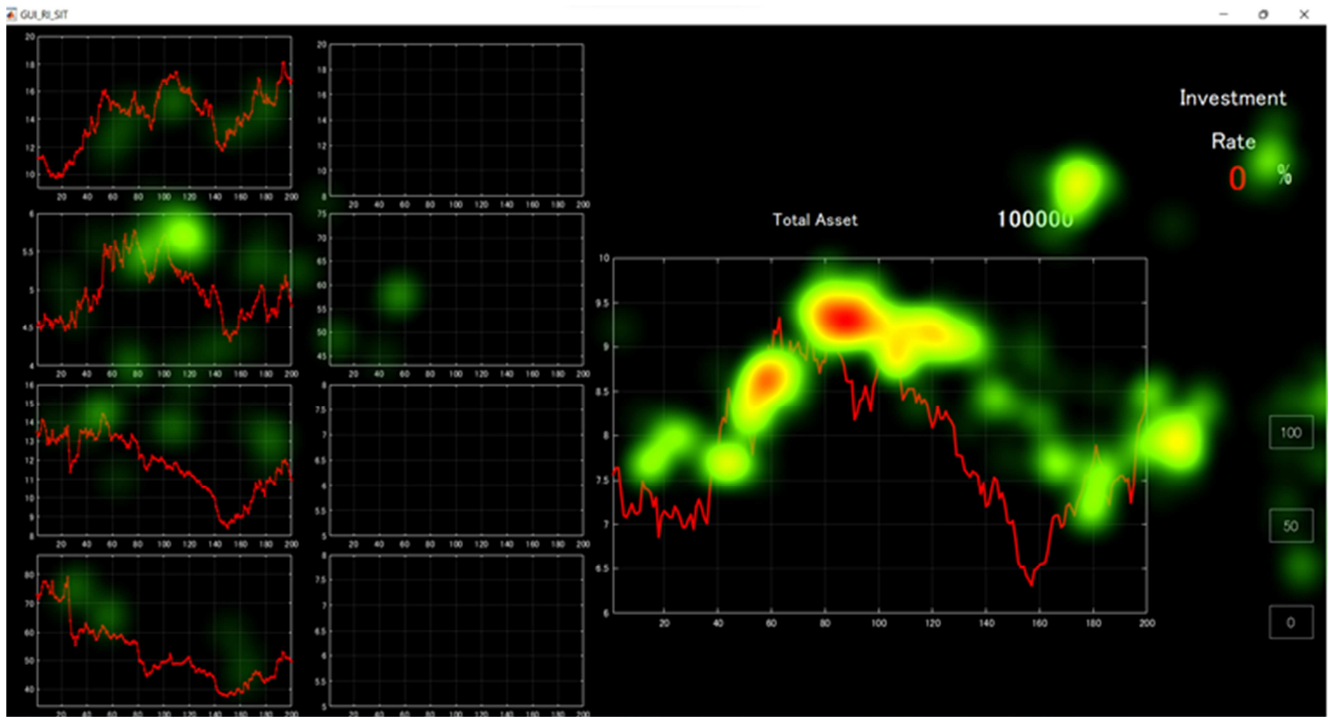


Figure 10. Heat map signal 4 case.

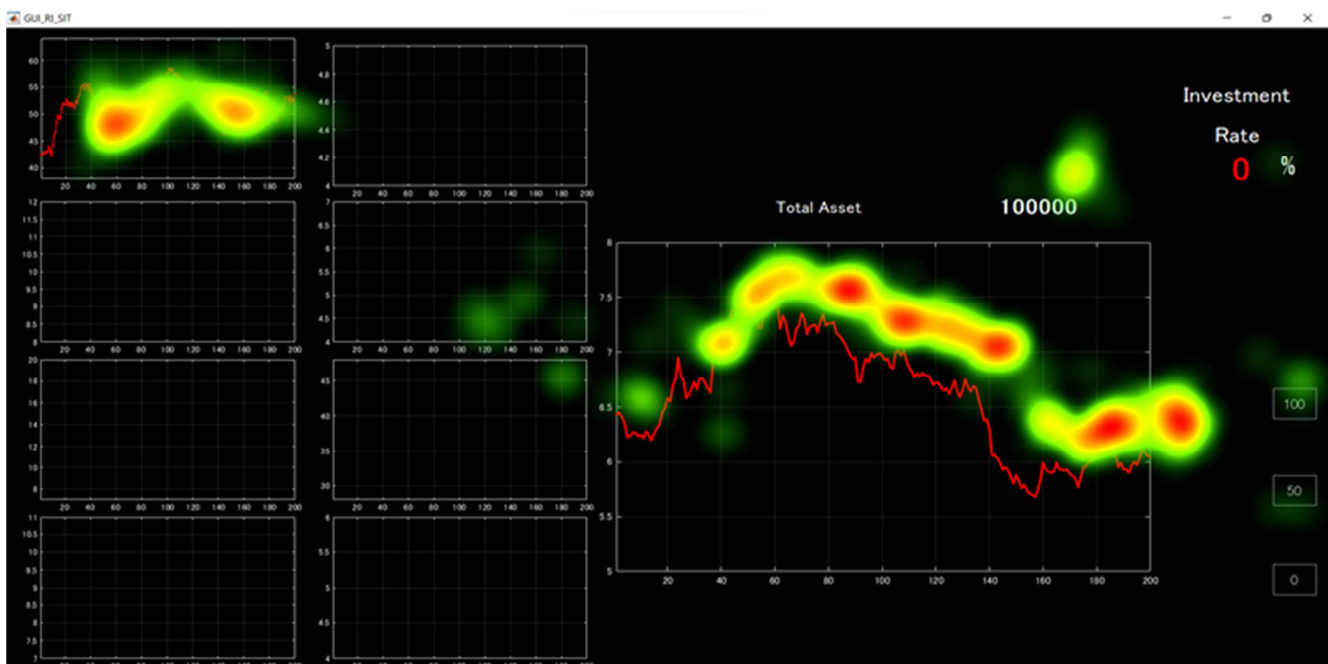


Figure 11. Heat map signal 1 case.

## 4. Conclusion

In this paper, we have examined the correspondence between the amount of mutual information and cognitive cost from a biometric perspective. Specifically, examined the consistency between behavior, changes in cerebral blood oxidized hemoglobin concentration in the dorsolateral prefrontal cortex, and Stochastic choice type model information cost parameter. And examined the consistency

of gaze information and capacity constraints of the KF model.

Our analysis showed that:

- (1) The stochastic choice RI model fit the behavioral data of the present experiment and that the cost parameter  $\lambda$  of the stochastic choice type model was significantly positively correlated with the activation status of the rostral prefrontal cortex and dorsolateral prefrontal cortex.
- (2) Demonstrated the consistency between gaze

information and the capacity constraint of the KF model, as expected, when there is a lot of information, not all information can be processed, so more accurate decisions cannot be made.

The cognitive cost represented by the amount of mutual information employed in the RI model is consistent with the activation of brain regions associated with cognitive cost, and thus indirectly supports the assumption of the RI model.

## References

- [1] Christopher A. Sims: Implications of rational inattention. *Journal of Monetary Economics* 50 (3), 665–690 (2003).
- [2] Filip Matejka and Alisdair McKay: Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review* 105 (1), 272-298 (2015).
- [3] Christopher A. Sims: Rational Inattention: Beyond the Linear-Quadratic Case. *American Economic Review* 96 (2), 158-163 (2006). doi: 10.1257/000282806777212431.
- [4] Filip Matejka: Rigid pricing and rationally inattentive consumer. *Journal of Economic Theory* 158, 656-678 (2015). doi: 10.1016/j.jet.2015.01.021.
- [5] D'Esposito, M., Detre, J. A., Alsop, D. C., et al.: The neural basis of the central executive system of working memory. *Nature* 378 (6554), 279-281 (1995). doi: 10.1038/378279a0.
- [6] Smith, Edward E., and John Jonides: Neuroimaging analyses of human working memory. *Proceedings of the National Academy of Sciences* 95 (20), 12061-12068 (1998). doi: 10.1073/pnas.95.20.12061.
- [7] Smith, Edward E and Jonides, John: Working memory: A view from neuroimaging. *Cognitive psychology* 33 (1), 5-42 (1997). doi: 10.1006/cogp.1997.0658.
- [8] Gupta, Rashmi, and Tranel, Daniel: Memory, neural substrates. *Encyclopedia of human behavior*, 593-600 (2012) DOI: 10.1016/B978-0-12-375000-6.00230-5.
- [9] Germann, J. and Petrides, M: Area 8A within the posterior middle frontal gyrus underlies cognitive selection between competing visual targets. *Neuro* 7 (5), (2020) doi: 10.1523/ENEURO.0102-20.2020.
- [10] Hamada Hamid: Networks in Mood and Anxiety Disorders. *Neuronal Networks in Brain Function, CNS Disorders, and Therapeutics* 327-334 (2014) doi: 10.1016/B978-0-12-415804-7.00024-1.
- [11] Yang Ming: Coordination with flexible information acquisition. *Journal of Economic Theory* 158, 721-738 (2015). doi: 10.1016/j.jet.2014.11.017.
- [12] Ravid, Doron: Bargaining with rational inattention. Available at SSRN 2957890 (2015). doi: 10.2139/ssrn.2957890.
- [13] Daniel Martin: Strategic pricing with rational inattention to quality. *Games and Economic Behavior* 104, 131-145 (2017). doi: 10.1016/j.geb.2017.03.007.
- [14] Mark Deanyand Nathaniel Nelighz: Experimental tests of rational inattention (2017). doi: 10.7916/d8-4w4k-3q85.
- [15] Ambuj Dewan and Nathaniel Neligh: Estimating information cost functions in models of rational inattention. *Journal of Economic Theory* 187, 105011 (2020). doi: 10.1016/j.jet.2020.105011.
- [16] Jakub Steiner, Colin Stewart, Filip Matějka: Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica* 85 (2), 521-553 (2017). doi: 10.3982/ECTA13636.
- [17] Bartosz Mackowiak, Filip Matejka, Mirko Wiederholt: Rational Inattention: A Review. ECB Working Paper, No. 2570 (2021). DOI: 10.2866/417246.
- [18] Bertoli S, Moraga J F H, Guichard L: Rational inattention and migration decisions. *Journal of International Economics* 126 103364 (2020). DOI: 10.1016/j.jinteco.2020.103364.
- [19] Caplin A, Dean M, Leahy J: Rational inattention, optimal consideration sets, and stochastic choice. *The Review of Economic Studies*, 86 (3): 1061-1094 (2019). DOI: 10.1093/restud/rdy037.
- [20] Lin Y H: Stochastic choice and rational inattention. *Journal of Economic Theory*, 202, 105450 (2022). DOI: 10.1016/j.jet.2022.105450.